CHAPTER VII

CORRELATION CURVES OF RETROACTIVELY FILTERED DATA

by

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SUMMARY

A concept for retroactively filtering a correlation curve is discussed and the results given. Normally, the data are filtered before being correlated with the net result that, if the limits you have set for your high and low pass filters are not suitable, the data must be completely reprocessed. Since the data must be either digitized or processed through a digital filter and then recorrelated, the time lost is considerable. Besides, when the data have been reprocessed, there is no assurance that the filters are still correctly set. Further, this process tends to make filtering of data in near-real-time wind measurements of limited value.

This chapter demonstrates the use of retroactive filtering to produce quickly filtered correlation curves from unfiltered data. This is done by Fourier transforming the correlation curves into the spectra, choosing the frequency range of interest (omitting any instrument noise frequencies), and then Fourier-transforming the spectra back into the time domain.

NOMENCLATURE

INDEPENDENT VARIABLES

Symbol	Definition
t	total observation time
T	total integration time

Symbol	Definition
ΔT	length of time for each piece -- the whole data record is subdivided into pieces for data processing
i	piece number
m	total number of pieces of data integrated over ($m = T/\Delta T$)
f	frequency
τ	time lag between the two signals
τ_m	maximum time lag

DEPENDENT VARIABLES

$x(t)$	signal time history for a sensor
$y(t-\tau)$	delayed time history for a second sensor
$r_i(\tau)$	covariance of the ith piece of data at time lag τ
$\overline{r_i(\tau)}$	accumulative covariance average over the first ith piece of data
$\overline{\overline{r(\tau)}}$	accumulative covariance average over the entire integration time
$\Delta r(\tau)$	is the statistical error of the covariance
$C(f)$	co-spectra (real part)
$Q(f)$	quad-spectra (imaginary part)
$G(f)$	gain, $G^2 = C^2 + Q^2$
$\langle G(f) \rangle$	mean gain over m pieces of data
$\overline{\overline{G(f)}}$	accumulative gain over m pieces of data
$\langle \Delta G(f) \rangle$	mean statistical error of the gain
$\overline{\overline{\Delta G(f)}}$	accumulative statistical error of the gain

$r_{G,\phi}$ — the Fourier transpose of the spectra to the correlation curve will have the subscripts of those spectra components used

$r_{yx}(\tau) = r_{xy}(-\tau)$ — this is the cross-covariance (subscript xy indicates positive time lags and yx indicates negative time lags

OPERATORS

$( )$ — statistic

$\overline{( )}_i = \frac{1}{\Delta T} \int_{(i-1)\Delta T}^{i\Delta T} ( ) \, dt$ — piecewise mean (note: when average done over real time or time lag, the symbol $< >$ is used

$\langle ( )_k \rangle = \frac{1}{k} \sum_{i=1}^{k} ( )_i$ — accumulative mean where k is the kth piece of data (when average over pieces)

$\overline{\overline{( )}} = \langle \overline{( )}_{k=m} \rangle$ — total accumulative mean

$\langle ( )_k \rangle^2 = \frac{1}{k-1} \sum_{i=1}^{k} (( )_i - \overline{( )}_k)^2$ — statistical error

$\langle ( ) \rangle = F \{ \langle ( ) \rangle \}$ — the symbols used in the time domain will be transferred into the frequency domain showing the origin of the terms

VII.1 FORMULATION

The concept of retroactive filtering is straightforward. The correlation curve is Fourier-transformed into the frequency domain so that the unwanted frequencies can be removed. Once these frequencies are removed, the spectra are then Fourier-transposed back into the time domain so that the correlation curve no longer contains the undesired frequency components that the original curve had.

The original covariance curve is obtained by

$$\overline{\overline{r_{xy}(\tau)}} = \frac{1}{N} \int_{o}^{T} [x(t) - \langle x \rangle][y(t-\tau) - \langle y \rangle]\, dt. \tag{1}$$

This curve is then Fourier-transposed into the frequency domain by considering the co-spectra $\overline{\overline{C(f)}}$ and quad-spectra $\overline{\overline{Q(f)}}$ individually:

$$\overline{\overline{C(f)}} = \int_{o}^{\tau_m} [\overline{\overline{r_{xy}(\tau)}} + \overline{\overline{r_{yx}(\tau)}}] \cos 2\pi f\tau\, d\tau, \tag{2}$$

and

$$\overline{\overline{Q(f)}} = \int_{o}^{\tau_m} [\overline{\overline{r_{xy}(\tau)}} - \overline{\overline{r_{yx}(\tau)}}] \sin 2\pi f\tau\, d\tau. \tag{3}$$

It follows that the modulus of the gain $\overline{\overline{G(f)}}$ is

$$\overline{\overline{G(f)}} = \sqrt{\overline{\overline{C(f)}}^2 + \overline{\overline{Q(f)}}^2}\ , \tag{4}$$

and the phase angle is

$$\overline{\overline{\phi(f)}} = \arctan \frac{\overline{\overline{Q(f)}}}{\overline{\overline{C(f)}}}\ . \tag{5}$$

The calculations are performed by the computer program given in Appendix I.

The transpose of the spectrum back into the time domain is given by

$$\underset{\overline{\overline{G}}\ \overline{\overline{\phi}}}{r}(\tau) = \int_{f_{min}}^{f_{max}} \overline{\overline{G(f)}} \cos [2\pi f\tau + \overline{\overline{\phi(f)}}]\, df. \tag{6}$$

The frequencies integrated over in equation (6) set the reactive filter. For example, if the reactive filter is used to set a band pass, only those frequencies within the desired band pass are transposed back into the time domain; i.e., f_{min} and f_{max} are set at the upper and lower limits of the band pass. This means basically that the correlation curve arrived at by this practice contains only those frequencies within a given band pass. A second form of filtering is sometimes desired: notch filtering, which is used to remove a very narrow band pass from the data. With the reactive filter, a narrow band pass is removed by not integrating over these frequencies.

The next section will examine how the concept of retroactive filtering works in practice. This can be performed using the computer program given in Appendix II.

VII-2. RESULTS

The general procedure of retroactive filtering on a correlation curve is summarized from data with a broad band pass (0.01 to 4.0 cps).

The first case to be considered is the use of retroactive filtering to set the band pass. The original correlation curve contains frequencies up to 4.0 cps. As can be seen in figure VII.1a, there is a great deal of noise on this curve. The noise frequencies are estimated to be between 1.0 and 2.0 cps. Figure VII.1b shows what happens when the upper frequency is reduced from 4.0 cps to 2.0 cps. The net result is that the correlation curve is almost unchanged, implying that the region from 2.0 to 4.0 cps had little effect on the curve. (This curve will be used as the basis for the notch filtering.)

When the upper frequency limit was set to 1.0 cps, the curve in figure VII.1c resulted. One can observe how well the retroactive filtering has removed the noise on the curve.

A second example of limiting the band pass of the correlation curve by reactive filtering is shown in figure VII.2. The top correlation curve has an upper frequency limit of 4.0 cps, and when the upper limit is reduced to 1.0 cps, the bottom correlation curve resulted. It is of interest to note how, in this case, reactive filtering actually improved the large peak in the correlation curve.

A second type of filtering commonly used is notch filtering. When the data are digitized, certain frequencies are introduced in this process -- if the signal level is low -- that cannot be eliminated by analog filtering. Retroactive filtering is a means of eliminating this noise.

In the first case (Figure VII.1) the noise was eliminated by the "brute force" method of limiting the upper frequency to just below the digitization noise. The disadvantage with this approach is that the information from frequencies greater than 1.0 cps is lost. Thus, ideally we would like just to notch-filter the digitization noise. The top curve in Figure VII.3 was notch-filtered in the three digitization noise frequencies; i.e., those three frequencies were removed. The bottom curve resulted. By increasing the bandwidth of these notch filters from 0.01 cps to 0.03 cps, the digitization noise can be completely removed.

VII-3. CONCLUSIONS

Retroactive filtering has smoothed the correlation curve. The primary advantages to retroactive filtering are that it can be done on small computers without reprocessing the data and it saves turn-around time. Retroactive filtering is ideal for studies in the regions of the spectrum which contribute to transit time measurements.

The principal disadvantage to reactive filtering is that harmonics of the noise will still remain in the results unless they are also removed, although in practice the harmonic effect can never be completely eliminated. In spite of this disadvantage, it is still believed that retroactive filtering is of great value in correlation data analysis and can be used in link 3 of the MULTCOR program to make cross-beam analysis faster and more effective.

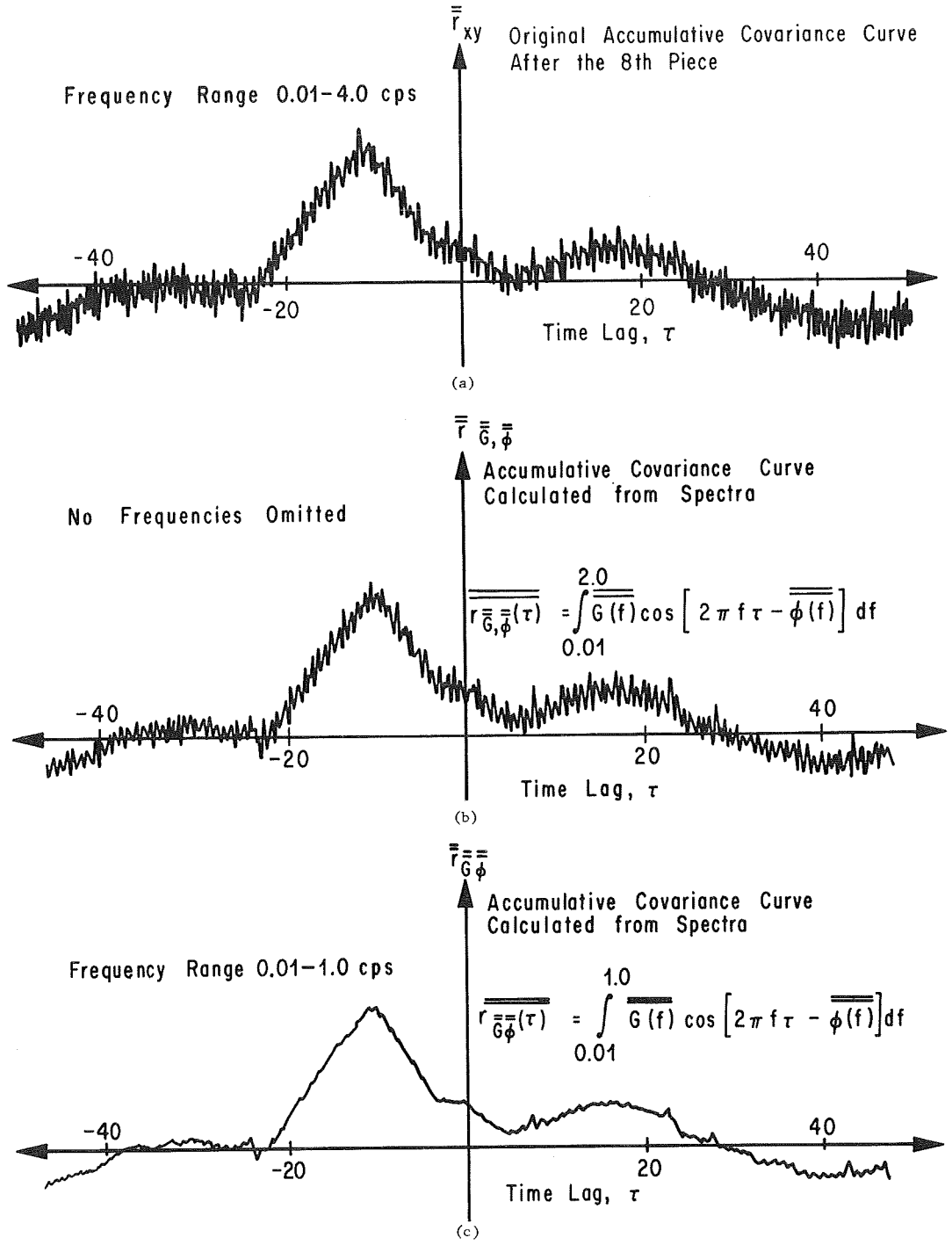

FIGURE VII.1. DIRECT AND DOUBLE-TRANSFORMED ACCUMULATIVE CORRELATION CURVES

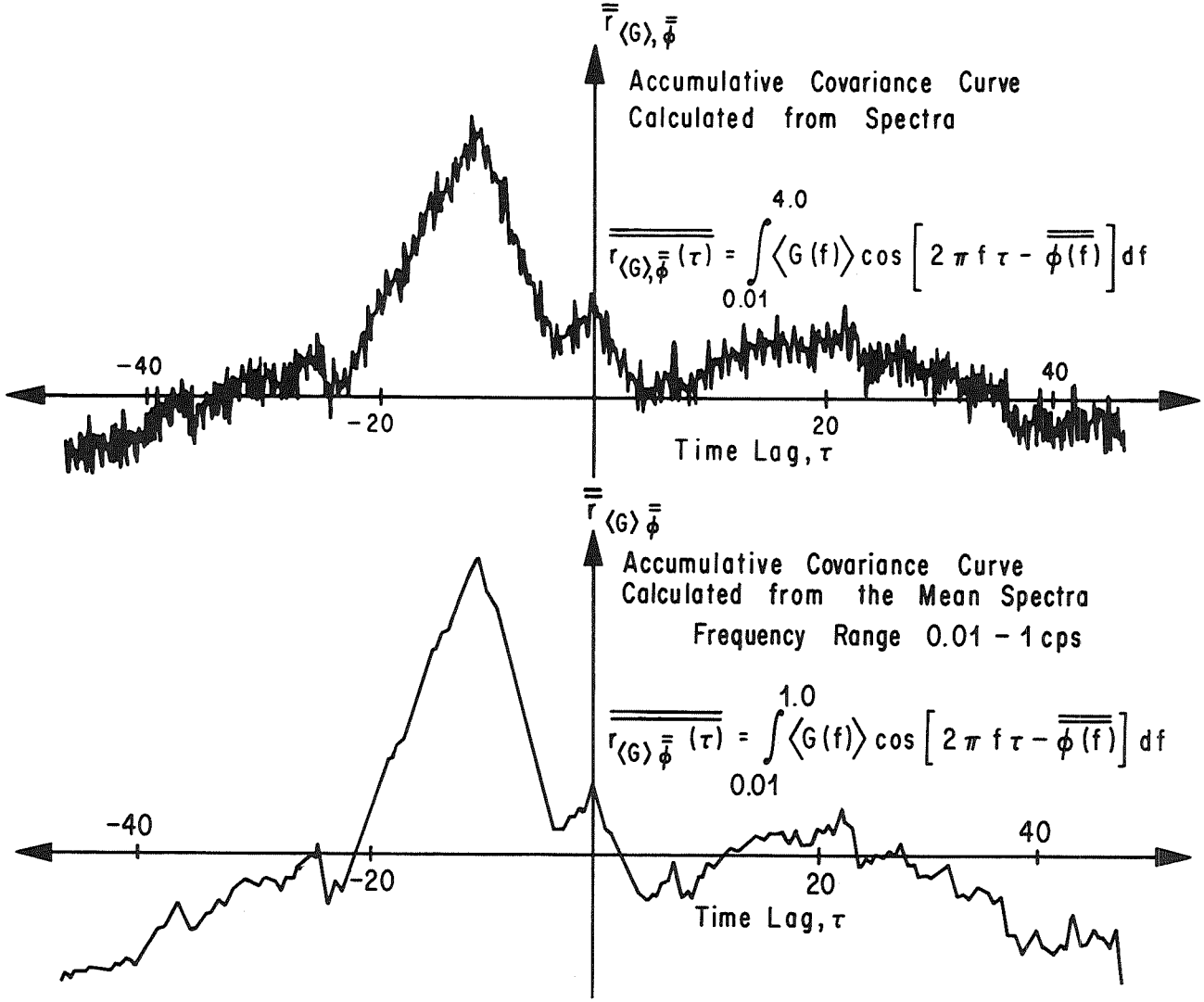

FIGURE VII.2. DIRECT AND DOUBLE-TRANSFORMED ACCUMULATIVE CORRELATION CURVES

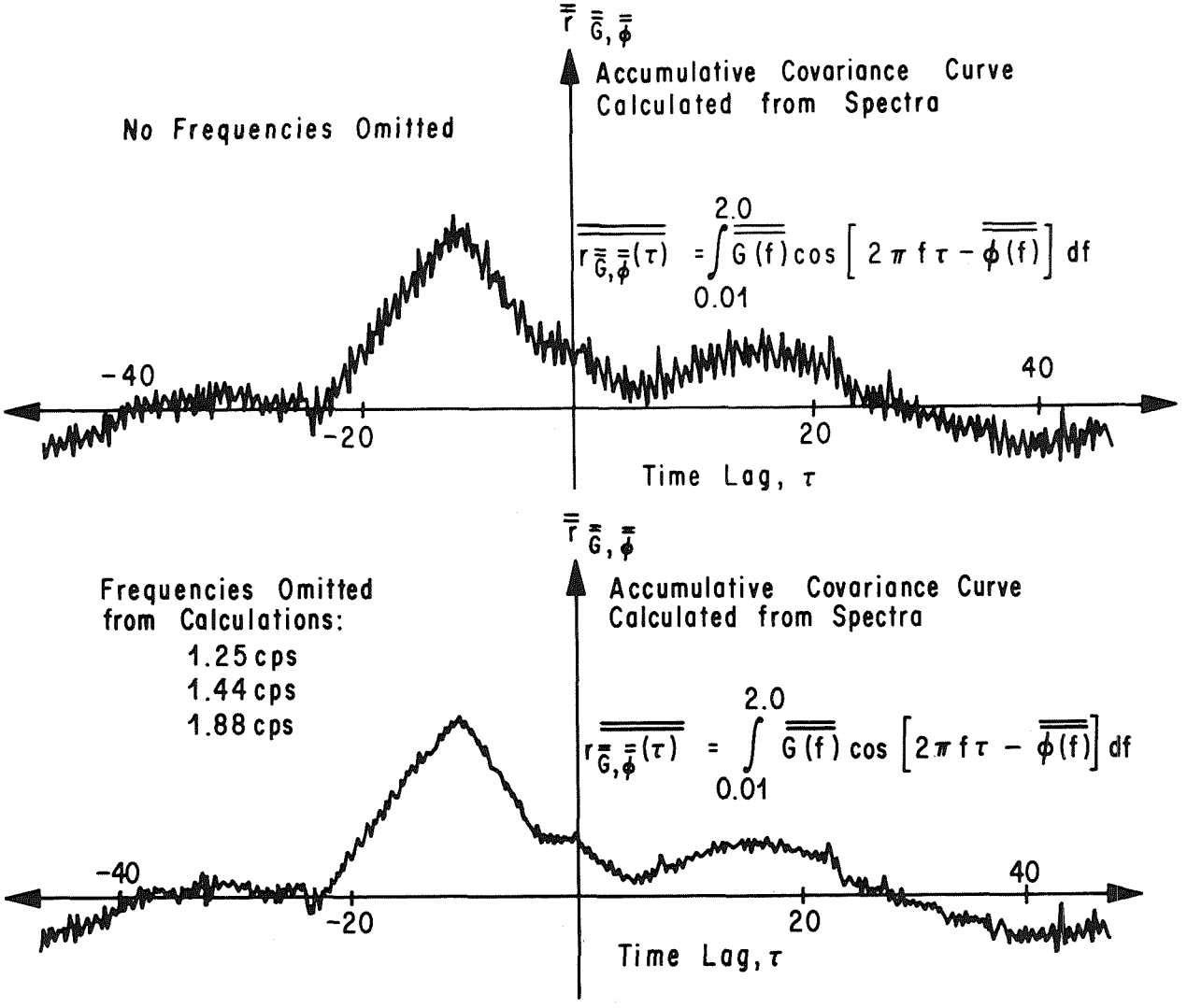

FIGURE VII.3. DIRECT AND DOUBLE-TRANSFORMED ACCUMULATIVE CORRELATION CURVES